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Analysis of cellulose networks by the finite element method

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Paper can be regarded as a network of cellulosic fibres, especially at lower basis weights. When the elastic behaviour of paper sheets is modelled, it is normally essential to know or to assume how the stresses (and strains) are distributed at the fibre level. This article presents an attempt to estimate how the stresses are transferred throughout a simple fibre network using the finite element method (FEM). Attention is mainly focused on the axial fibre stress distribution when the network is uniaxially deformed. The presence of fibre ends is found to induce local stress increases ("stress concentrations") in the deformed network, which presumably have a bearing on the ultimate properties of the sheet. The influence of the properties of the bonds between crossing fibres on the mechanical properties is also investigated. It is noted that the bond stiffness has no significant effect on the stress transfer between fibres provided that the stiffness is above a critical value. Below this value the stress transfer deteriorates rapidly.

1. Introduction

Paper is composed of a large number of cellulose fibres which are bonded together to form a sheet. The fibres themselves are mechanically anisotropic and are furthermore usually oriented to some extent in the machine direction of the web. The paper sheet is consequently also anisotropic (orthotropic). The sheet properties are, however, determined not only by the fibre properties but also by the number of fibres per unit volume (the sheet density). The fibre geometry is also of considerable importance. Paper may thus be regarded as a fibre network, especially at lower densities. The network character of paper is clearly demonstrated in Fig. 1, which is a micrograph of a low basis weight paper.

It is not surprising to find that several models which have been proposed to predict the mechanical properties of paper are "network models" [1–5]. The goal of any such theory is, of course, to relate the macroscopic properties, in the first place the elastic properties, of the paper to the geometrical arrangement of the fibres constituting the network and to the corresponding properties of the fibres themselves. Owing to the complexity of the problem, it is natural that any model used must be based on a rather idealized foundation. Nevertheless, the majority of the network theories apparently model the macroscopic behaviour of paper in a reasonable way, cf. also [6]. However, at the actual fibre level, the stresses and strains are known to an appreciably lesser extent. This is true of, for example, the axial stress distribution (or strain distribution) in a given fibre in the deformed network and the stress transport between the fibres. The effect of finite fibres (or equivalently the fibre ends) and the influence of the properties of the bonds between crossing fibres are also of significance when the stress distribution at the "microlevel" is discussed. Further knowledge of this kind is certainly required if a detailed understanding of the mechanical behaviour of cellulosic networks is sought for. Such information is particularly valuable with regard to the post-elastic behaviour of the network.

In many network theories it is assumed that the fibre strain is related to the sheet strain in a simple way, i.e. by a geometrical relation. In other words, that the individual fibres deform in the same way as the network itself. This is not necessarily true

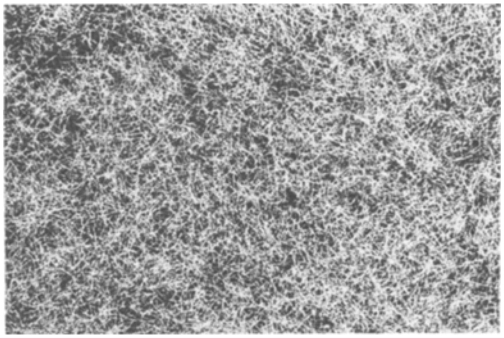

Figure 1 Photomicrograph of a cellulosic network (paper sheet) of low basis weight (~ 8 g m^{-2}).

and this is one of the items discussed in this article. Another important question is: how is the applied load transferred from fibre to fibre in the network? Obviously the crossing fibres play an important role in this connection in the same way as does the matrix material in a short-fibre reinforced composite, in which a load is transferred to the fibres via shear stresses in the interface region between fibre and matrix. In the case of a fibre network the properties of the bond between two crossing fibres in the network are thus important. The properties of the bond material are of special relevance for dry-formed (or air-formed) sheets formed in the absence of water, which results in a lack of bonding between the cellulose fibres in the network [7]. The stress transfer through these networks is poor and this is also the case with the mechanical properties. To overcome this drawback, small amounts (10 to 20% by weight) of a polymeric binder are added. This binder, to a first approximation, increases the stiffness and strength of the bond between the fibres and this results in an improvement of the property profile [7]. The properties of the bond are thus of the utmost importance for this particular group of cellulose-based composite materials. In many of the current network theories, however, the adhesion between the individual fibres is assumed to be perfect, i.e. the properties of the bond do not enter explicitly into the analysis. The influence of the bond properties on the stress distribution in the network deserve to be studied in more detail.

1.1. Objectives of the present work

The primary aims of the present work can be summarized as being:

1. to analyse the axial stress distribution in cellulose fibres in the network with special regard to the fibre ends and the number of fibres per unit volume (the sheet density);

2. to investigate the influence of the bond stiffness on the stress (or strain) transfer between different fibres.

Considering the complexity of the fibrous network, cf. Fig. 1, it is obvious that any attempt to quantify the stress distribution in the network must start from rather simple models. The model used here is no exception to this and it admittedly contains a number of imperfections. It should be regarded as a first attempt to outline the capability of the finite element method (FEM) to analyse problems of the types indicated above. It is believed that an analysis of this kind is valuable when a deeper insight into the network mechanisms is required. It may also provide a starting point for more detailed investigations using FEM.

The FEM-program used in this work is the advanced general purpose program ADINA [8]. It is assumed that all fibres are straight and isotropic and the analysis is restricted to the elastic regime. These restrictions are not imposed by the FEM-program itself but are a reflection of the present lack of experimental knowledge relating to the fibre properties. The analysis can be extended to include orthotropic fibres and elastic–plastic behaviour, but such an extension cannot be regarded as meaningful at this stage.

2. Finite element modelling of the network (two-dimensional analysis)

2.1. The network model

The network model used in the FEM-analysis must be highly idealized. Fig. 2 shows the FEM-mesh used to study the effect of uniaxial deformation in the x-direction of a network model. The network model consists of a number of fibres of finite length oriented in the direction of the applied load, these fibres being connected by perpendicularly crossing fibres. The adhesion between two fibres crossing each other is assumed to be perfect. The influence of the bond properties will be discussed in a later section of this paper. The network used here is apparently simple, but is rather similar to the structural element used by Perkins [5] for modelling network structures. The fibres are depicted as straight lines, but in the analysis the fibre width is assumed to be 30 μm and the thickness 3 μm. The total number of fibres is 94 and the fibre length (of the horizontal

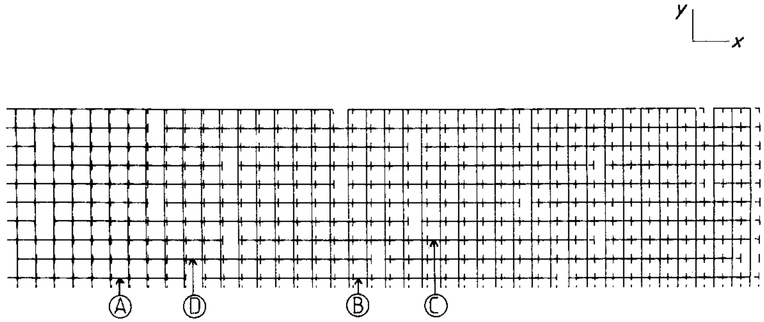

Figure 2 The FEM model of a network used for analysing the axial fibre stress distribution. The load is applied in the x-direction. The small vertical lines indicate the displacements of the fibre crossings when a strain of 1.3% is applied.

fibres) varies between 0.18 and 1.8 mm. The network is assumed to be symmetrical with regard to the in-plane co-ordinate axis, i.e. Fig. 2 shows only one quadrant of the entire network. The fibres are modelled by beam elements (2-nodes) which can also take into account shearing deformations. Non-linear geometrical displacements are allowed for in the analysis. The fibres are assumed to be mechanically isotropic with an elastic modulus of 20 GPa [6].

The density of the two-dimensional network shown in Fig. 2 is 750 kg m^{-3} obtained from a geometrical consideration assuming that the density of the cellulose fibres is 1500 kg m^{-3}. For comparison an analysis of a network with a density of 500 kg m^{-3} is also included. The small vertical lines in Fig. 2 denote the displacements of the fibre crossings when a deformation of ~ 1.3% (with respect to the total length of the network) is applied.

2.2. The axial fibre stress distribution

In the network shown in Fig. 2, three fibres have been denoted A, B and C. Fig. 3 shows the axial fibre stresses on these fibres (network density 750 kg m^{-3}) when a strain of 1.3% is applied to the network. Close to the fibre ends, the axial stress (and strain) is low. This is analogous to the behaviour of short-fibre reinforced plastics [9, 10] and merely indicates that fibre ends are not very effective in transferring stresses through the network. The axial stresses increase rather sharply with increasing distance from the ends and tend to reach a plateau level. If, however, a neighbouring fibre has an end beside the fibre under consideration, this will give rise to a relatively sharp peak in the stress distribution, cf. Fig. 3. If there are two separate fibre ends close to the given fibre, two peaks in the axial stress distribution are obtained as in the case of fibre C. If there are no fibre ends close to the fibre, the plateau stress level will be maintained over the major part of the fibre length (with the exception of its own ends). Fig. 3 shows that the presence of neighbouring fibre ends causes an increase in the axial stress of ~ 40% compared with the plateau level.

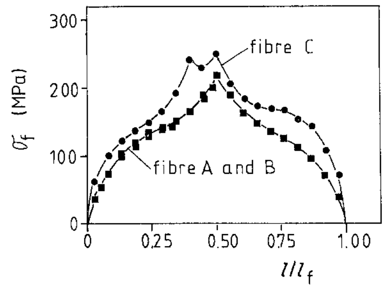

Figure 3 The axial fibre stress distributions for fibres A, B and C in Fig. 2. The x-axis is normalized with regard to the fibre length (l_f) of each fibre. The stress distribution for fibre A is assumed to be symmetrical with regard to its fixed end.

The existence of these stress peaks due to neighbouring ends is naturally of great significance for the mechanical properties of paper structures, especially for the onset of rupture. Fewer fibre ends would have a beneficial influence on the strength. The cause of the appearance of the stress peaks is here simply the requirement of load equilibrium. The load carried by a finite fibre close to its end must be redistributed to the next fibres. A breakage in a fibre in the network would have the same effect. For short-fibre reinforced composites, the importance of accounting for the fibre ends, which are responsible for stress concentrations in the polymeric matrix, has previously been emphasized [11, 12].

The effect of sheet density on the axial stress distribution is shown for fibre A in Fig. 4. The stress distributions shown here correspond to a network strain of 1%. If the sheet density is reduced from 750 kg m^{-3} to 500 kg m^{-3}, the axial stress level becomes somewhat lower, i.e. the

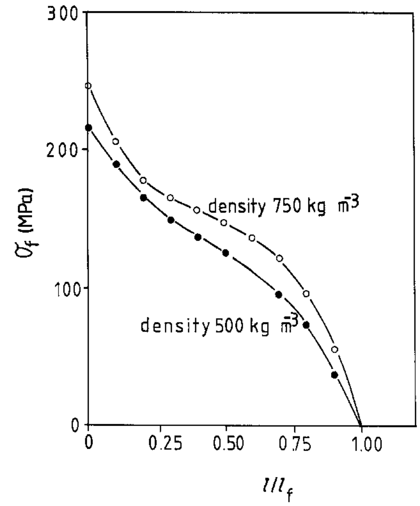

Figure 4 The effect of sheet (network) density (ρ_s) on the axial fibre stress distribution for fibre A in Fig. 2.

stress transfer is less effective. The reason for this is discussed in some detail below, but it is mainly due to a decrease with decreasing density of the number of crossing fibres (bond sites), which transfer the stress between the horizontal fibres in the structure. A dense network is thus stiffer than one of lower density, which is certainly in agreement with experimental findings [5, 6]. The stress concentrations increase in magnitude however as the density increases.

2.3. Comparison between network strain and fibre strain

Owing to the network structure, the total strain in the network, i.e. in the sheet, is not transformed to the same magnitude of strain at the fibre level. By fibre strain we here mean the elongation of the fibre as a whole divided by its length. The actual axial strain varies of course along the fibre length in the same way as does the axial fibre stress. The difference between the network strain and the fibre strain is a result of the bending of the crossing fibres (vertical fibres) close to the end of a horizontal fibre, which leads to a translation of the fibre segments in the vicinity of the ends rather than in an elongation. To this may be added the usual inability of the fibre ends to bear a load [5]. The calculated displacements along the length of the fibres A, B and D (in Fig. 2) are shown in Fig. 5. These are neighbouring fibres. The displacements are shown for two sheet densities; 500 and 750 kg m^{-3}. The total applied network strain (ϵ_t) is in this case 1.3%. It is evident in Fig. 5 that the displacements of fibres A and B differ considerably from that of fibre D although they are neighbours (vertically). This is due to the presence of the fibre ends. The difference decreases with increasing sheet density, owing to the higher bond site density (number of crossing fibres) at higher sheet densities, which results in an improved stress (or strain) transfer between the fibres. A higher bond site density decreases the bending of the crossing (vertical) fibres.

As already mentioned, the longitudinal fibres are not strained to the same extent as the network itself. At a sheet density of 500 kg m^{-3}, the calculated ratio of the fibre strain (ϵ_f) to the network strain (ϵ_t) is only 0.63 for fibre A. For fibre D, which is longer and for which the translation of the fibre ends is, in a relative sense, less important, the ratio ϵ_f/ϵ_t is higher: 0.80. Still the fibres are not axially deformed to the same extent as the network itself. If the sheet density is increased

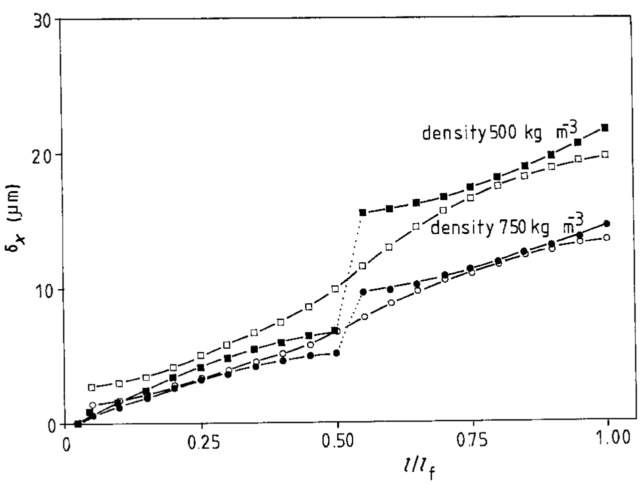

Figure 5 The displacements along the fibre length for fibres A, B and D in Fig. 2. The upper curves refer to $\rho_s = 500$ kg m^{-3} and the lower to $\rho_s = 750$ kg m^{-3}. The filled symbols refer to fibres A and B and the unfilled to fibre D. The x-axis is normalized with regard to the length (l_f) of fibre D.

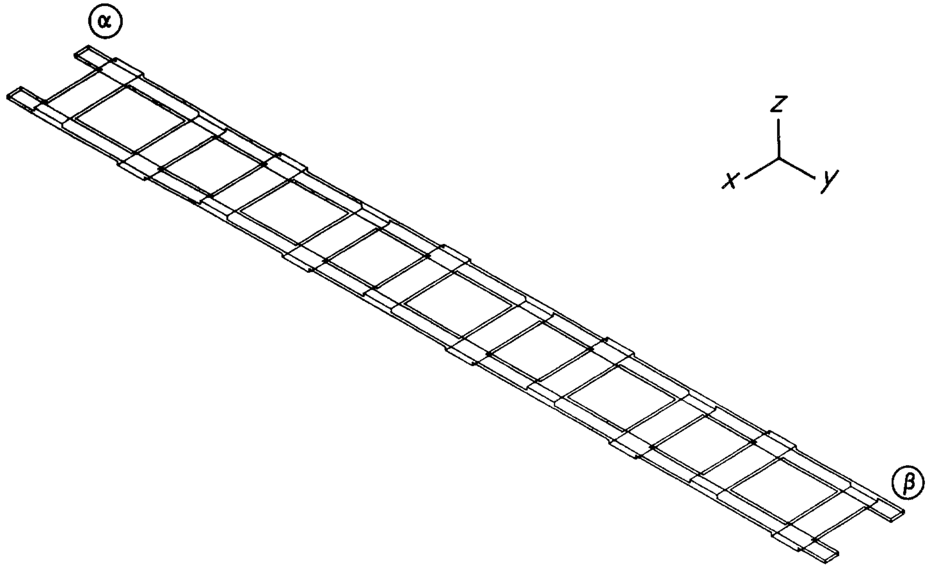

Figure 6 The FEM model used for analysing the influence of the bond stiffness on the stress transfer between two fibres ($\rho_s = 500$ kg m^{-3}).

to 750 kg m^{-3} the ϵ_f/ϵ_t-ratio increases somewhat. For example, for fibre A to 0.73. Consequently, in a less dense network of the type studied here, the fibre strains in the load direction will be lower although the total network strain may be the same. This is also the reason why the axial fibre stresses are lower at lower densities, cf. Fig. 4.

In some network theories it is assumed that the fibre strains may be evaluated in a straightforward manner from the network strain. The present FEM-analysis indicates that this may be difficult due to the movement of the fibre ends. This is certainly a problem which will require more attention in future work. In passing, it can be mentioned that the axial stress distribution obtained here is formally rather similar to that assumed for short-fibre reinforced composites [9, 10], i.e. the axial stress (or strain) decreases sharply near the fibre ends. In networks, however, the movements of the fibres relative to each other provide an additional complication.

For completeness, it should be mentioned that the specific tensile stiffness, i.e. the elastic modulus divided by the sheet density, of the network shown in Fig. 2 is ~ 10 kNm g^{-1} at a sheet density of 500 kg m^{-3}. This corresponds to an elastic modulus of ~ 5 GPa.

The network model used here is only suitable for studying the uniaxial behaviour of the fibres. If the properties in the direction transverse to the applied load are also to be studied in detail, a more refined model is required. For example, the Poisson's ratio of the network in Fig. 2 is close to zero, which is to be expected for a model of this type [13]. This problem can be circumvented by introducing additional beam elements with different bending and tensile stiffnesses [13]. This is, however, beyond the scope of the present analysis.

3. Influence of the bond stiffness on the stress transfer between fibres

In this section the influence of the stiffness of the bond between crossing fibres on the stress transfer between two adjacent fibres is discussed. The finite element model is shown in Fig. 6 and in more detail in Fig. 7 where the bond material between the fibres is indicated. The fibres are modelled using three-dimensional isoparametrical elements (12 to 16 nodes). The fibres have a length of 1.8 mm and the structure shown in Fig. 6 corresponds to a network density of 500 kg m^{-3}.

The upper fibre in Fig. 6 is uniaxially deformed (in the fibre direction) and the transferred stresses and strains in the fibre direction of the lower fibre are evaluated. It should be mentioned that the end denoted α of the upper fibre is not fixed (completely free). In a series of analyses the deformation of the upper fibre was chosen to be the same as that of fibre D in Fig. 2; i.e. the lower fibre in Fig. 6 corresponds to fibre A in Fig. 2. In this way the results of the previous analysis were used as

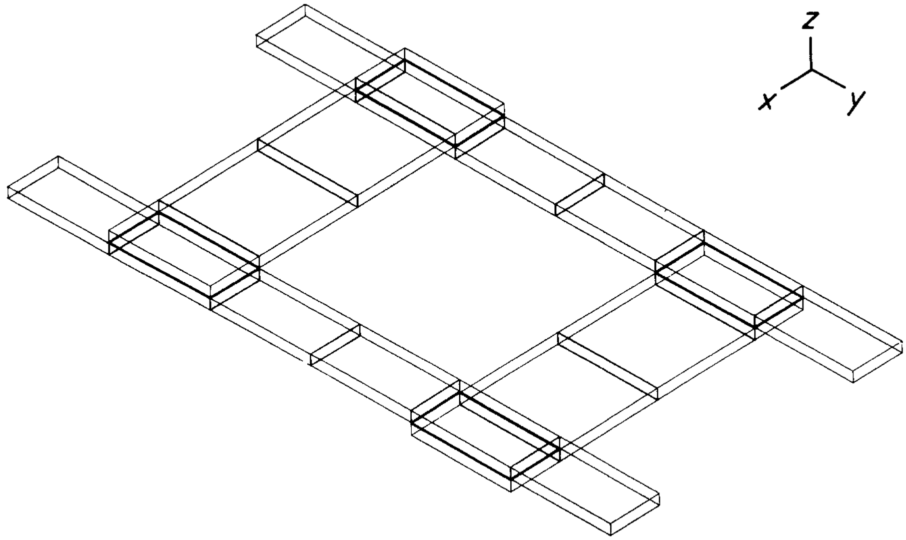

Figure 7 Close-up of Fig. 6. The bonding elements between the crossing fibres are indicated.

boundary conditions in the present study. The axial stress distribution in the lower fibre in Fig. 6 did not then significantly differ from that obtained for fibre A in Fig. 2. However, in the major part of the analysis described in this section the fibre end α was free and the end β was displaced a given amount. Allowing the fibre end α to be free slightly increases the stress peak in the axial stress distribution of the lower fibre, cf. Fig. 4, but this has no influence on the results here presented.

Since there is no easily accessible information regarding the bond stiffness, we have here chosen to vary it between 1 MPa and 20 GPa. In a way this simulates how the bond stiffness increases as more and more polymeric binder is added to a dry-formed network of cellulosic fibres or, of course, how binders of different stiffnesses affect the stress transfer when the binder content is kept constant. A typical soft type of binder used for dry-formed products has a stiffness of ~ 1 to 10 MPa. The thickness of the bond is chosen to be 0.3 μm (1/10 of the fibre thickness).

Fig. 8 shows the ratio of the strain in lower fibre, ϵ_f (transferred strain) to the total strain ϵ_t, applied to the model in Fig. 6 as a function of the bond stiffness, E_b, given as log (E_b/E_f) where E_f is the fibre modulus, 20 GPa. The strain transferred from the upper fibre to the lower is not markedly affected by the bond stiffness even down to such a low value as ~ 50 MPa. However, below this level the transferred strain decreases sharply. The insensitivity of the elastic properties in the fibre direction for short-fibre reinforced composites when the bond stiffness is changed moderately has also been noted by Agarwal and Bansal [14] using a similar FEM-technique.

The corresponding axial stress distributions in the lower fibre are shown in Fig. 9. If the bond stiffness is above ~ 50 MPa, it apparently has no great influence on the transferred stresses. Decreasing the bond stiffness down to 1 MPa leads, however, to a dramatic deterioration in the stress transfer between the fibres. A bond stiffness of this magnitude will presumably yield a structure with a poor mechanical property profile.

For dry-formed papers the situation is in a sense reversed. The stress transfer between the fibres is initially poor. When small amounts of a

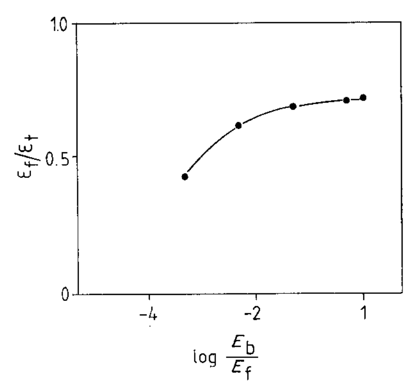

Figure 8 The ratio of the strain in the lower fibre (ϵ_f), i.e. the transferred strain, to the total strain (ϵ_t) applied plotted against log (E_b/E_f) where E_b is the bond stiffness and E_f the fibre modulus.

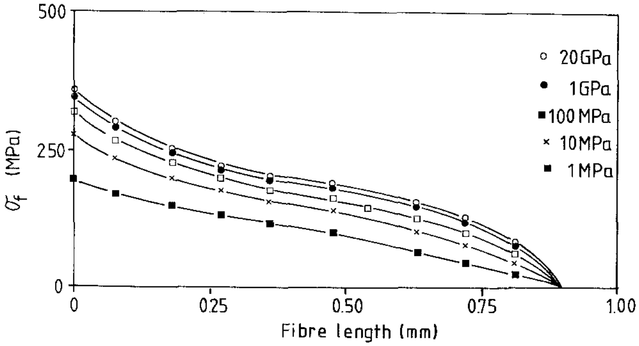

Figure 9 The influence of the bond stiffness (E_b) on the axial stress distribution of the lower fibre in Fig. 6.

synthetic binder are added, the bond stiffness is correspondingly increased. This results in a rather large improvement in the stress transfer between the fibres and thus in the mechanical properties. A further increase in binder content is, however, in a relative sense less efficient. This behaviour is in qualitative agreement with experimental findings [7].

As a final point it can be mentioned that the stresses are transferred from the upper to the lower fibre by bending of the crossing fibre segments. The corresponding bending stresses are highest in the crossing fibre segments near the fibre ends (of the two longitudinal fibres). The maximum magnitude of the stresses in the crossing segments is about half the maximum axial fibre stress.

4. Final remarks

This work should be regarded as an attempt to analyse the fibre stresses and strains in a paper network using the finite element method. The model is a very simple one and is primarily aimed at describing the axial stress distributions. Nevertheless, some useful indications are found with the FEM technique. First, the effect of the fibre ends is very pronounced in the stress distributions. This is analogous to the situation encountered with short-fibre reinforced composites [11, 12]. This may indicate that the problem of predicting the strength of paper may be studied along lines similar to those used by Halpin and Kardos [11]. The existence of such stress peaks is undoubtedly of importance with regard to the ultimate properties of paper.

The reduction of the fibre strain compared with the total strain, which is also an effect of the finite fibre length, is noteworthy. The FEM analysis indicates that this should result in a lower stiffness, especially at lower sheet densities. It might be possible to improve existing network models by incorporating this effect. To some extent, the reduction in fibre strain could explain why low density paper has a rather low modulus despite the fact that the cellulose fibres are rather stiff in the fibre direction, cf. also [6].

It is also found that the bond stiffness does not significantly affect the stress transfer between the fibres and thus does not influence the elastic properties of the network unless it is rather low. However, below a "critical" value the stress transfer decreases sharply. The behaviour of a dry-formed network of cellulose fibres when polymeric binder is added is qualitatively described by this change in stress transfer, as already has been noted. There are several more aspects of adhesion and bond properties and their effect on the network properties which might be analysed by the FEM technique. One example is the effect of fibre–fibre friction on the stress distributions in the network.

To gain more knowledge concerning the network properties, more refined models than the one employed here, can be used. It would also be interesting to analyse the elastic–plastic behaviour and the effect of orthotropic elastic fibres. However, this requires more basic experimental work concerning the properties of the fibres themselves. When such information is at hand, FEM has the potential of being a useful tool for advancing fundamental understanding of network mechanics.

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